Effect of Electron-Electron Interactions on the Conductivity of Clean Graphene

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Minimal conductivity of a single undoped graphene layer is known to be of the order of the conductance quantum, independent of the electron velocity. We show that this universality does not survive electron-electron interaction, which results in nontrivial frequency dependence. We begin with analyzing the perturbation theory in the interaction parameter g for the electron self-energy and observe the failure of the random-phase approximation. The optical conductivity is then derived from the quantum kinetic equation, and the exact result is obtained in the limit when $g \ll 1 \ll g|\ln\omega|$.

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Introduction.—Recent experiments on transport in graphene layers [1–3] have validated extensive theoretical efforts directed at understanding various properties of two-dimensional Dirac fermions. A lot of this effort is devoted to the zero temperature dc conductivity which has a universal value of the order of the conductance quantum [4,5]. Such a value is not unexpected from the dimension analysis since the intrinsic (undoped) graphene lacks a characteristic momentum scale. Two different minimal conductivities are conventionally defined. The dc limit of an ac conductivity in clean graphene ($\tau^{-1} = 0$, $\omega \to 0$) was shown to be $\sigma = e^2/4\hbar$ [6]. Another possible definition of a strict dc limit of impure graphene ($\omega = 0$, $\tau^{-1} \to 0$) gives a different, but numerically close, value $\bar{\sigma} = 2e^2/\pi^2\hbar$ [6]. Recent calculations have largely confirmed [7–13] the results of Ref. [6], while others obtained different values [14]. Conductivity in bilayer graphene has also been a subject of close theoretical attention [15–18].

It is noteworthy that the minimal conductivity σ is very much analogous to the optical conductivity of a two-dimensional electron system with spin-orbit-split band structure, where it is due to the "chiral resonance," and the corresponding value $e^2/16\hbar$ [19,20] is exactly 4 times smaller than σ (which is the degree of spin-nodal degeneracy in graphene). This analogy is due to the similarity in the chiral properties of the eigenstates in the two systems.

In this Letter, we demonstrate that the notable universality of the values of σ and $\bar{\sigma}$ does not hold in the presence of electron-electron interactions, which results in a strong frequency dependence of the conductivity. Here we concentrate on the optical conductivity $\sigma(\omega)$ in strict disorder-free graphene ($\tau^{-1} = 0$) and show that the optical conductivity is actually *suppressed* by interactions in comparison with its "universal" value.

A single intrinsic 2D graphene layer is described by the chiral Hamiltonian

$$H = v_0 \sum_{i\mathbf{p}} \hat{c}^{i\dagger}_{\mathbf{p}} \hat{\boldsymbol{\sigma}} \cdot \mathbf{p} \hat{c}^{i}_{\mathbf{p}} + \frac{1}{2} \sum_{ij\mathbf{pkq}} \hat{c}^{i\dagger}_{\mathbf{p}-\mathbf{q}} \hat{c}^{j\dagger}_{\mathbf{k}+\mathbf{q}} V_{\mathbf{q}} \hat{c}^{j}_{\mathbf{k}} \hat{c}^{i}_{\mathbf{p}}, \quad (1)$$

where "hats" denote operators in pseudospin space ($\hat{\boldsymbol{\sigma}}$ represents the usual set of Pauli matrices); the sum over Latin indices is taken over two nodal points and two (true) spin directions. The interaction potential is $V_{\mathbf{q}} = 2\pi e^2/\kappa|\mathbf{q}|$, with κ being the dielectric constant of a substrate, and v_0 is the "bare" electron velocity. Hereinafter we denote $\sum_{\mathbf{p}} \equiv \int d^2p/(2\pi)^2$ and set $\hbar = 1$ throughout the text, except in the final result (20).

Let us begin with the perturbation theory, in powers of the dimensionless interaction constant $g = e^2/\kappa v_0$, for the electron self-energy (at $T = 0$). We find that the random-phase approximation (RPA) *fails* for the system described by the Hamiltonian (1), as the non-RPA contributions are generally not small.

Perturbation theory.—The first- and second-order interaction corrections to the electron self-energy are shown in Fig. 1. The solid line corresponds to the electron Green's function

$$\hat{G}_{\epsilon\mathbf{p}} = \frac{1}{\epsilon - v_0 p \hat{\sigma}_{\mathbf{p}} + i\eta \mathrm{sgn}\epsilon} = \frac{1}{2} \sum_{\beta=\pm1} \frac{1 + \beta\hat{\sigma}_{\mathbf{p}}}{\epsilon - \beta(v_0 p - i\eta)}, \quad (2)$$

where $\hat{\sigma}_{\mathbf{p}} = \hat{\boldsymbol{\sigma}} \cdot \mathbf{n}$ is the projection of the pseudospin operator onto the direction of the electron momentum $\mathbf{n} = \mathbf{p}/p$. The first-order contribution [Fig. 1(a)] is independent of the energy variable

$$\hat{\Sigma}^{a}_{\mathbf{p}} = i \sum_{\mathbf{p}'} \int \frac{d\epsilon}{2\pi} V_{\mathbf{p}-\mathbf{p}'} \hat{G}_{\epsilon\mathbf{p}'} = \frac{1}{2} \hat{\sigma}_{\mathbf{p}} \sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'} (\mathbf{n} \cdot \mathbf{n}'). \quad (3)$$

The integral here diverges logarithmically at $p' \gg p$. This divergence is cut off at the inverse lattice spacing $\mathcal{K}$, leading to the logarithmic correction

$$\hat{\Sigma}^{a}_{\mathbf{p}} = p\hat{\sigma}_{\mathbf{p}} \frac{e^2}{4\kappa} \ln(\mathcal{K}/p). \quad (4)$$

Such logarithmic renormalization of the electron velocity was first discussed in Ref. [21] within the scope of the renormalization group approach.

The second-order correction consists of three contributions [Figs. 1(b)–1(d)], of which the last one is identically zero. This fact can be understood upon noticing that the

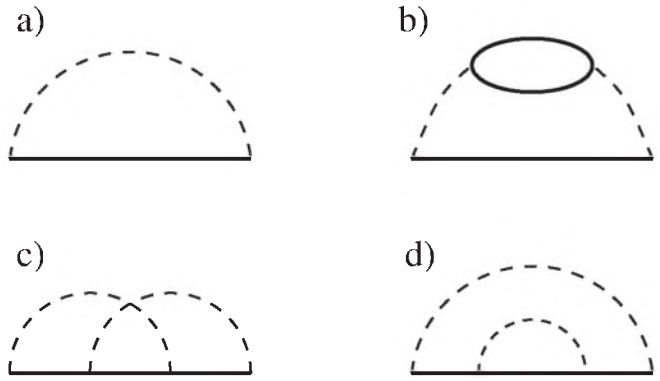

FIG. 1. (a) First- and (b),(c),(d) second-order corrections to the electron self-energy. The solid line represents the electron Green's function (2); the dashed line stands for the bare interaction potential $V_{\mathbf{q}}$. The closed loop implies summation over nodal and spin degeneracy.

diagram 1(d) is responsible for the renormalization of the Fermi level [22] and is absent in clean graphene due to electron-hole symmetry. The other two graphs give nonzero contributions. The RPA-type diagram [Fig. 1(b)] gives

$$\hat{\Sigma}^{b}_{\epsilon\mathbf{p}} = 4i\sum_{\mathbf{q}}\int\frac{d\omega}{2\pi}V_{\mathbf{q}}^2\hat{G}_{\epsilon-\omega,\mathbf{p}-\mathbf{q}}\Pi(\omega,q), \tag{5}$$

where the coefficient 4 comes from the summation over the nodal points and (real) spin states in the internal loop, which represents the polarization operator

$$\Pi(\omega,q) = -i\mathrm{Tr}\sum_{\mathbf{p}'}\int\frac{d\epsilon'}{2\pi}\hat{G}_{\epsilon'+\omega,\mathbf{p}'+\mathbf{q}}\hat{G}_{\epsilon'\mathbf{p}'}; \tag{6}$$

here the trace is taken in the pseudospin space. To evaluate the integrals, it is convenient to use the following integral representation for the polarization operator:

$$\Pi(\omega,q) = \frac{q^3 v_0}{8\pi}\int_0^\infty\frac{dx}{\omega^2 - q^2v_0^2(1+x^2) + i\eta}. \tag{7}$$

The integral over frequency in Eq. (5) can now be easily calculated. Restricting for simplicity to the $\epsilon = 0$ case here [23], we obtain

$$\hat{\Sigma}^{b}_{\mathbf{p}} = -\frac{\pi e^4}{\kappa v_0}\sum_{\mathbf{q}}\int_0^\infty\frac{dx}{\sqrt{1+x^2}}\frac{\hat{\sigma}_{\mathbf{p}-\mathbf{q}}}{q\sqrt{1+x^2}+|\mathbf{p}-\mathbf{q}|}. \tag{8}$$

The main (logarithmic) contribution to this integral comes from large values of transferred momentum q. Expanding to the linear order in p/q and evaluating first the integral over the angle between $\mathbf{p}$ and $\mathbf{q}$, then the integral over dx, and finally over the absolute value of q, we arrive at

$$\hat{\Sigma}^{b}_{\mathbf{p}} = -p\hat{\sigma}_{\mathbf{p}}\frac{e^4}{6\kappa v_0}\ln(\mathcal{K}/p). \tag{9}$$

The last contribution [Fig. 1(c)] is given by ($\epsilon = 0$)

$$\hat{\Sigma}^{c}_{\mathbf{p}} = \frac{1}{4v_0}\sum_{\mathbf{p}'\mathbf{p}''}V_{\mathbf{p}-\mathbf{p}'}V_{\mathbf{p}-\mathbf{p}''} \times\frac{\hat{\sigma}_{\mathbf{p}'}+\hat{\sigma}_{\mathbf{p}''}-\hat{\sigma}_{\mathbf{p}'+\mathbf{p}''-\mathbf{p}}-\hat{\sigma}_{\mathbf{p}'}\hat{\sigma}_{\mathbf{p}'+\mathbf{p}''-\mathbf{p}}\hat{\sigma}_{\mathbf{p}''}}{p'+p''+|\mathbf{p}'+\mathbf{p}''-\mathbf{p}|}. \tag{10}$$

The leading logarithmic contribution into this expression comes from large p', $p'' \gg p$. To this leading order, it is sufficient to expand the integrand to the linear power in $\mathbf{p}$. As the relevant terms appear from the interaction potentials as well as from the expansion of both the numerator and the denominator, the calculations are too cumbersome to be presented here. Finally,

$$\hat{\Sigma}^{c}_{\mathbf{p}} = p\hat{\sigma}_{\mathbf{p}}\frac{e^4}{\kappa v_0}\left(-\frac{2}{3}+\ln 2\right)\ln(\mathcal{K}/p). \tag{11}$$

The expressions (4), (9), and (11) determine the renormalization of the electron velocity up to the second order in the electron-electron interaction

$$\frac{v_p}{v_0} = 1 + \left[\frac{g}{4} - g^2\left(\frac{5}{6}-\ln 2\right) + O(g^3)\right]\ln(\mathcal{K}/p). \tag{12}$$

Two important conclusions can be drawn from this result.

(i) The contribution (11) from the non-RPA diagram with intersecting interaction lines *is not parametrically small* compared with the RPA term (9). Resulting in an overestimation of about ~20% in the second order, the neglect of non-RPA corrections to the electron self-energy becomes an uncontrollable approximation in the higher orders in g for undoped graphene. Our findings, thus, do not support the conjecture of Ref. [24] (where it was used for the calculation of the electron lifetime) that RPA is an exact approximation in the limit $g \ll 1$. Experimentally, the value of the velocity is $v_0 = 1.1\times 10^6$ m/s [2]. For a typical dielectric substrate $\kappa \approx 6$, which yields $g \approx 0.3$, indicating that $g \ll 1$ is a reasonable approximation.

(ii) It is essential that the higher orders *do not produce* higher powers of $\ln(\mathcal{K}/p)$. This is most simply verified for the RPA terms [21]. On the other hand, the diagrams with intersecting interaction lines are not more singular than the RPA ones, as is clear from the power counting (but also not parametrically smaller than RPA terms). As the consequence, in the limit $g \ll 1$ it is sufficient to restrict to the lowest order of the perturbation theory. Still, the product $g\ln(\mathcal{K}/p)$ *can be arbitrarily large* without violating the perturbation expansion.

Kinetic equation.—We now apply the obtained understanding to the analysis of the ac conductivity $\sigma(\omega)$. Free-electron conductivity follows directly from the polarization operator (7) and the particle conservation condition. Taking into account nodal and spin degeneracy,

$$\sigma(\omega) = 4\lim_{q\to 0}\frac{ie^2\omega}{q^2}\Pi(\omega,q) = \frac{e^2}{4}. \tag{13}$$

In order to calculate the homogeneous ac conductivity in the presence of electron-electron interactions, we apply a quantum kinetic equation. This will allow us to take into

account the leading terms $\sim g\ln(\mathcal{K}/p)$ in the exact way (but still assuming $g \ll 1$). The density matrix $\hat{f}_{\mathbf{p}}$ should be defined as a matrix in a pseudospin space $f_{\mathbf{p}}^{\alpha\beta} = \langle\langle c_{\mathbf{p}}^{\beta\dagger} c_{\mathbf{p}}^{\alpha}\rangle\rangle$, where α and β are the pseudospin indices. Using the equations of motion for the operators $\hat{c}_{\mathbf{p}}$ determined by the Hamiltonian (1), and assuming a homogeneous in space (but time-dependent) external electric field $\mathbf{E}$, it is straightforward to write a closed equation for the density matrix $\hat{f}_{\mathbf{p}}$ to the linear order in the electron-electron interactions:

$$\frac{\partial \hat{f}_{\mathbf{p}}}{\partial t} + iv p[\hat{\sigma}_{\mathbf{p}}, \hat{f}_{\mathbf{p}}] + e\mathbf{E}\cdot\frac{\partial \hat{f}_{\mathbf{p}}}{\partial \mathbf{p}} = i\sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'}[\hat{f}_{\mathbf{p}'}, \hat{f}_{\mathbf{p}}]. \tag{14}$$

We emphasize that the kinetic equation for graphene (14) does contain the terms linear in the interaction. This is in contrast to the conventional system with parabolic dispersion for which nontrivial contributions (collision integral) appear in the second order in $V_{\mathbf{q}}$ [25]. This difference is due to the chiral structure of the electron eigenstates in graphene. The kinetic equation needs to be solved to the linear order in the applied electric field $\hat{f}_{\mathbf{p}} = \hat{f}_{\mathbf{p}}^{(0)} + \hat{f}_{\mathbf{p}}^{(1)}$ around the equilibrium distribution $\hat{f}_{\mathbf{p}}^{(0)} = 1 - \hat{\sigma}_{\mathbf{p}}/2$ (at zero temperature). The contribution resulting from the term $[\hat{f}_{\mathbf{p}'}^{(0)}, \hat{f}_{\mathbf{p}}^{(1)}]$ represents simply the electron self-energy $\Sigma_{\mathbf{p}}^{a}$ to the first order in the interaction. The remaining term $[\hat{f}_{\mathbf{p}'}^{(1)}, \hat{f}_{\mathbf{p}}^{(0)}]$ describes vertex corrections. For the periodic electric field, the linearized kinetic equation takes the form

$$-i\omega\hat{f}_{\mathbf{p}}^{(1)} + iv_p p[\hat{\sigma}_{\mathbf{p}}, \hat{f}_{\mathbf{p}}^{(1)}] = \frac{e}{2p}(\mathbf{E}\times\mathbf{n})\cdot(\hat{\boldsymbol{\sigma}}\times\mathbf{n}) - \frac{i}{2}\sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'}[\hat{f}_{\mathbf{p}'}^{(1)}, \hat{\sigma}_{\mathbf{p}}], \tag{15}$$

where the renormalized velocity v_p is given by the expression (12) taken to the first order in g. With the following substitution:

$$\hat{f}_{\mathbf{p}}^{(1)} = i(\mathbf{E}\times\mathbf{n})\cdot(\hat{\boldsymbol{\sigma}}\times\mathbf{n})A(p) + \mathbf{E}\cdot(\hat{\boldsymbol{\sigma}}\times\mathbf{n})B(p), \tag{16}$$

the matrix integral equation (15) reduces to a pair of coupled scalar integral equations:

$$\begin{aligned} \omega A(p) + 2v_p p B(p) &= \frac{e}{2p} + \sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'} B(p')\cos\theta_{pp'}, \\ 2v_p p A(p) + \omega B(p) &= \sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'} A(p')\cos^2\theta_{pp'}, \end{aligned} \tag{17}$$

where $\theta_{pp'}$ is the angle between the vectors $\mathbf{p}$ and $\mathbf{p}'$. The system of Eqs. (17) can be solved by means of the consecutive approximations in $V_{\mathbf{p}-\mathbf{p}'}$. Figure 2 illustrates diagrammatically the meaning of these consecutive orders. In the limit $g \ll 1$, it is sufficient to stop at the first-order term (see the discussion at the end of this section):

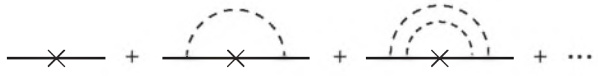

FIG. 2. Graphic representation of the density matrix given by the integral equation (15). The dashed line represents the interaction potential $V_{\mathbf{q}}$; the solid line stands for the electron propagator corrected by the self-energy $\Sigma_{\mathbf{p}}^{a}$. The cross denotes the applied external field. The first-order term brings a logarithmic contribution into the optical conductivity $\propto \int dp/v_p p$. The higher-order terms contain higher power of the small coupling constant g but do not result in higher powers of logarithms.

$$A(p) = \frac{e}{2p}\frac{\omega}{\omega^2 - 4v_p^2 p^2} - e\omega\sum_{\mathbf{p}'} V_{\mathbf{p}-\mathbf{p}'}\frac{v_p p\cos^2\theta_{pp'} + v_{p'}p'\cos\theta_{pp'}}{p'(\omega^2 - 4v_p^2 p^2)(\omega^2 - 4v_{p'}^2 p'^2)}. \tag{18}$$

Here, as usual, the frequency should be understood as possessing the infinitesimally small positive imaginary part $\omega \to \omega + i\eta$. Knowledge of the function $A(p)$ allows one to determine the electric current $\mathbf{j} = 4e\,\mathrm{Tr}\sum_{\mathbf{p}} v_0 \hat{f}_{\mathbf{p}}\hat{\boldsymbol{\sigma}} = \sigma(\omega)\mathbf{E}$, which yields the conductivity

$$\sigma(\omega) = \frac{2ei}{\pi}\int_0^{\infty} dp\, v_0 p A(p). \tag{19}$$

The real part of the conductivity is due to the zeros of the denominators of Eq. (18). In the second term in Eq. (18), the leading logarithmic contribution comes from the imaginary part that arises due to the second singularity ($|\omega| = 2v_{p'}p'$). It is important that the second-order term in the expansion equation (18), which is not explicitly written, *does not* yield any $g^2\ln^2 p$ contribution, being only $\sim g^2\ln p$, and, thus, by virtue of $g \ll 1$ resulting only in a small correction to the first term, similar to the situation already encountered in the self-energy [cf. Eq. (12)]. Upon extracting this imaginary part, one should first evaluate the integral over the angle $\theta_{pp'}$ and then over the momentum p. Straightforward calculation leads to the low-frequency optical conductivity

$$\sigma'(\omega) = \frac{e^2}{4\hbar}\frac{v_0}{v_\omega}\left[1 + \ln\left(\frac{v_\omega}{v_0}\right)\right], \tag{20}$$

where $v_\omega = v_0[1 + g\ln(v_0\mathcal{K}/|\omega|)/4]$ is the renormalized electron velocity, corresponding to frequency ω. In deriving Eq. (20), it is utilized that $\int_{\omega/v_0}^{\mathcal{K}} dp/v_p p = 4\ln(v_\omega/v_0)/gv_0$.

Equation (20) is the main result of this Letter. The conductivity depends *only* on the combination of the frequency and the coupling constant $g\ln(v_0\mathcal{K}/|\omega|)$. We illustrate the dependence of the optical conductivity on the frequency for different values of the coupling constant g in Fig. 3. When frequency increases, the optical con-

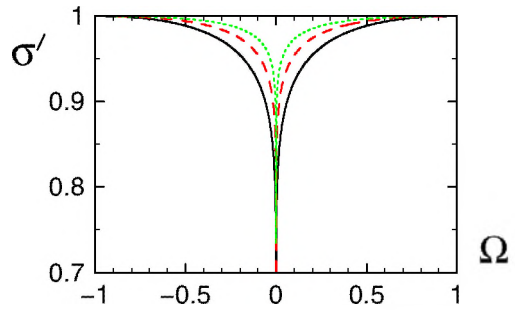

FIG. 3 (color online). The dependence (20) of the real part of the optical conductivity (measured in units of the noninteracting conductivity $e^2/4\hbar$) on the dimensionless frequency $\Omega = \omega/v_0\mathcal{K}$ for different values of the coupling constant: $g = 0.5$ (dotted green line), $g = 0.7$ (dashed red line), and $g = 1.0$ (solid black line). The suppression of the conductivity is pronounced in the low-frequency domain.

ductivity approaches its value $e^2/4\hbar$, but at lower frequencies its magnitude is significantly reduced.

Restricting the perturbation expansion (18) to the lowest-order vertex correction requires some justification. In particular, one has to make sure that this expansion does not lead to higher-order logarithms in the discarded terms, i.e., that no x-ray (Mahan) singularity [22] takes place. Indeed, the second-order contribution into $A(p)$ can be easily derived and analyzed. The corresponding expression is cumbersome, but the conclusion is straightforward. The higher-order terms still feature single-logarithmic divergencies but arrive with higher powers of g. The situation here is similar to the one already encountered above in the calculation of the electron self-energy; see Eq. (12).

Summary and conclusions.—Optical conductivity in undoped graphene is the result of the "chiral" resonance, i.e., resonant creation of an electron-hole pair. The chiral Hamiltonian leads to nonzero matrix elements of the velocity operator for the intersubband transitions, resulting in a finite conductivity in a homogeneous electric field [26].

Effects of electron-electron interaction are twofold. First, the logarithmic renormalization of the electron velocity at low energies leads to the *decrease* in the density of states, which *suppresses* the probability of intersubband transitions. This effect is accounted for by the electron self-energy and is revealed in the appearance of the factor v_0/v_ω in expression (20). Second, the excited electron and hole interact in the final state. This interaction is attractive and results in the relative *enhancement* of the optical conductivity. Such final-state interactions are analogous to the excitonic effect in conventional semiconductors and are accounted for by the vertex corrections (Fig. 2), yielding the factor $1 + \ln(v_\omega/v_0)$ into Eq. (20). Utilization of the quantum kinetic equations allows one to consider both of these effects on equal footing and account for the leading logarithmic terms in the limit when $g \ll 1 \ll g\ln(v_0\mathcal{K}/|\omega|)$. Note that, in order to be able to study homogeneous conductivity, one should be able to neglect charge accumulation at the boundaries and associated with its electric field. It is easy to estimate that the system size should be large enough, $L \gg gv_0/\omega$.

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